HW # 1 Solutions

(1) (2 pts) $v = f \cdot \lambda$

$\lambda = 5 \times 10^{-7}$ m

$f = 6 \times 10^{8}$ Hz

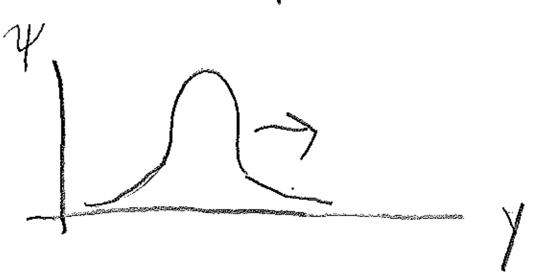
$v = 300$ m/s

(2) (8 pts, 2 pts for each part)

(a) $\Psi(y,t) = e^{-(ax')^2}$

if $x' = y - vt$ and $v = \frac{b}{a}$. ← speed

Yes ; positive y direction ; profile is gaussian :



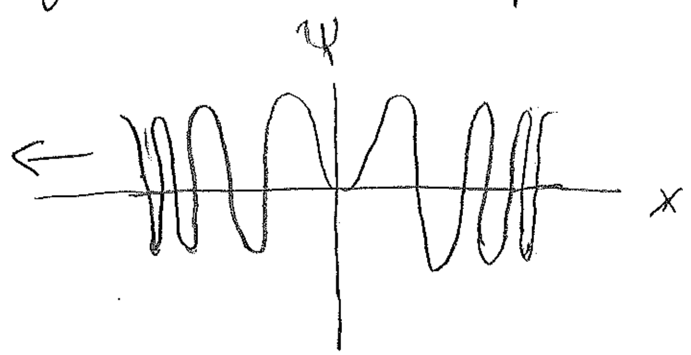
(b) $\Psi(z,t) = A \sin\left[(\sqrt{a}\, z + \sqrt{b}\, t)(\sqrt{a}\, z - \sqrt{b}\, t)\right]$

Not a traveling wave, because it cannot reduce to a single variable $x' = z - vt$ or $x' = z + vt$.

(c) $\Psi(x,t) = A \sin\left[\frac{2\pi}{a^2}\left(x + \frac{a}{b} t\right)^2\right]$

Yes; negative x direction; speed is $v = \frac{a}{b}$

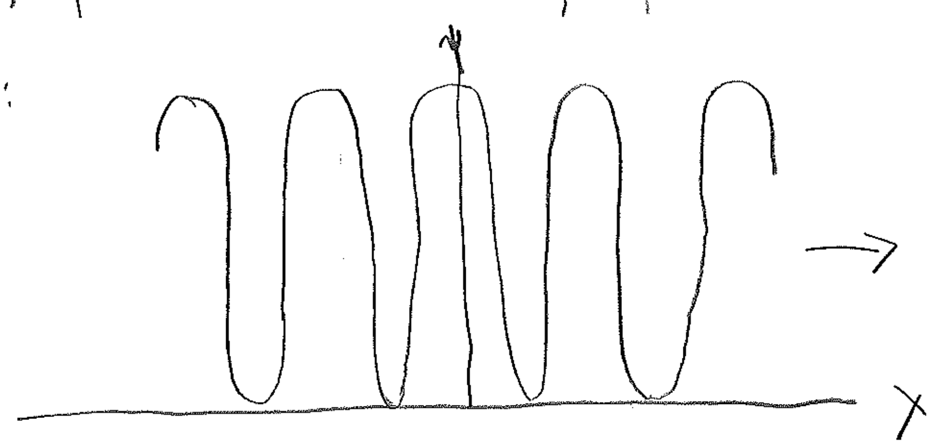
Profile:



(d) $\Psi(x,t) = A\left[\frac{1}{2} + \frac{1}{2}\cos\left(4\pi(t - x)\right)\right]$

Yes; positive x direction; speed is $v = 1$

Profile:



③ (3 pts)

(2.64 & 2.65) $\Psi(x,y,z,t) = f(\alpha x + \beta y + \gamma z \mp vt)$ ↙ or g

Let $x' = \alpha x + \beta y + \gamma z \mp vt$.

- $\frac{\partial}{\partial x}\Psi(x') = \frac{\partial f}{\partial x'} \cdot \frac{\partial x'}{\partial x} = \alpha \cdot \frac{\partial f}{\partial x'}$
- $\frac{\partial^2}{\partial x^2}\Psi(x') = \frac{\partial}{\partial x}\left[\alpha \cdot \frac{\partial f}{\partial x'}\right] = \alpha \frac{\partial}{\partial x'}\left[\frac{\partial f}{\partial x}\right]$

$$= \alpha \frac{\partial}{\partial x'}\left[\alpha \frac{\partial f}{\partial x'}\right] = \boxed{\alpha^2 \frac{\partial^2}{\partial x'^2} f(x')}$$

Similarly, for $y, z,$ and t :

- $\frac{\partial^2}{\partial y^2}\Psi(x') = \beta^2 \frac{\partial^2}{\partial x'^2} f(x')$
- $\frac{\partial^2}{\partial z^2}\Psi(x') = \gamma^2 \frac{\partial^2}{\partial x'^2} f(x')$
- $\frac{\partial^2}{\partial t^2}\Psi(x') = (\mp v)^2 \frac{\partial^2}{\partial x'^2} f(x')$

Substitute into 3D wave equation:

$$\frac{\partial^2}{\partial x^2}\psi + \frac{\partial^2}{\partial y^2}\psi + \frac{\partial^2}{\partial z^2}\psi = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\psi$$

$$\alpha^2 \frac{\partial^2}{\partial x'^2} f(x') + \beta^2 \frac{\partial^2}{\partial x'^2} f(x') + \gamma^2 \frac{\partial^2}{\partial x'^2} f(x') = \frac{1}{v^2} v^2 \frac{\partial^2}{\partial x'^2} f(x')$$

$$\alpha^2 + \beta^2 + \gamma^2 = 1 \quad \checkmark$$

(4) (3 pts)

$$\psi(x,t) = A\cos(kx - \omega t)$$

or

$$\psi(x,t) = A\sin(kx - \omega t)$$

or

$$\psi(x,t) = A e^{i(kx-\omega t)}$$

and you could use "$(kx - \omega t + \varepsilon)$" instead.

5 (4 pts)

θ	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\sin\theta$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0
$\sin(\theta-\frac{\pi}{2})$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1
sum	-1	$-\sqrt{2}$	-1	0	1	$\sqrt{2}$	1	0	-1	$-\sqrt{2}$	-1